# Two-Hop Communications in a Cognitive Radio Network 

Md. Mizanur Rahman, Jelena Mišić and Vojislav B. Mišić Ryerson University, Toronto, ON, Canada


#### Abstract

In this paper, we describe an efficient bridging protocol for interconnection of cognitive personal area networks (CPANs) and evaluate its performance through probabilistic analysis. We also take into account the impact of collisions with primary user transmissions through probabilistic analysis and renewal theory. As the bridge roams between CPANs at will (without predefined scheduling) the performance of both local and non-local traffic in either CPAN depends on the local and non-local traffic intensities. The results also show that the collision probability with primary source does not affect bridge performance at low to medium CPAN traffic.


## I. Introduction

Opportunistic spectrum access allows for more efficient spectrum sharing between licensed or primary users (PUs) and non-licensed, cognitive-capable secondary users (SUs) [2], [3]. To establish effective communications, SU devices or nodes form a Cognitive Personal Area Network (CPAN) under the control of a dedicated coordinator node. The coordinator allocates bandwidth to individual nodes upon request and makes decisions about switching channels, often done in a rapid, frequency-hopping manner [4] to avoid interference to and from PU transmissions. Channel switching decisions are typically made on the basis of spectrum sensing [5], [8], aided by static information about transmitter locations, frequencies, and hours of operation, where such information is available [1]. Overseeing spectrum sensing through allocating sensing tasks

[^0]to nodes and collecting the results of their sensing is thus another important task for the CPAN coordinator.

Multi-hop opportunistic spectrum access networks are beginning to attract research attention [6], [12], with performance of data transfers being among the most important research challenges. While the operating (coexisting) of Multiple Cognitive Radio Networks on the same channel through controlling transmission powers and sharing that channel in time division manner is discussed in [17], the procedure of data transfer between two such networks is not addressed. In our earlier work [13] we have considered unidirectional data transfer in a network formed by two channel-hopping CPANs, hereafter referred to as source and destination CPANs, respectively. The CPANs use the transmission tax-based MAC protocol [9] and a dedicated bridge node that switched back and forth between CPANs to deliver data from the source to the destination CPAN. However, the analysis was done under a number of assumptions: first, that CPAN superframes are synchronized so that they begin and finish at the same time; second, that the bridge switched from one to the other CPAN in each superframe; finally, that bridge transmissions were given higher priority by scheduling them early in the superframe.

In a subsequent paper [14], we have relaxed these constraints to allow CPANs superframes to be skewed in time by an arbitrary period and to allow the bridge to switch between the CPANs without any predefined arrangement, which results in simplified bridge scheduling and increased fairness for all nodes. Last but not least, we assume that the bridge carries bidirectional traffic, which is more feasible in practice.

The findings of data transmission between two asynchronous CPANs (a time difference between the starting of two CPANs superframes) in [15], and bidirectional traffic exchange between these two CPANs in [14] are not limited to two-hop networks. These can be applied to multi-hop networks. A multi-hop network can be formed when one or more CPANs are situating in between the source and destination CPANs and dedicated bridges are assigned to each pair of CPANs from source to destination CPAN. Therefore, the bridge which carries the bidirectional traffic between two asynchronous CPANs is suitable for multi-hop networks because in multi-hop network the CPANs are not synchronized and traffic need to be flown both ways.

In this paper, we extend our analysis to include the impact of collisions with primary source transmissions in the course of node (be it ordinary node or a bridge) transmission. We model the operation of the resulting two-hop network using probabilistic analysis and renewal theory,


Fig. 1. Bridge switching algorithm for two-way traffic
and obtain the probability distributions of CPAN and bridge cycle times as well as of end-to-end delays for both local and non-local (i.e., intra- and inter-CPAN) traffic. We also present the probability of collisions with primary source transmissions.

The rest of the paper is structured as follows: Section II describes the CPAN environment and the operation of the bidirectional bridge. Section III presents the probabilistic model of bridging algorithm with transmission, sensing and reception by ordinary nodes. Estimation of collision with primary source is presented in Section IV. Access delay for both intra-CPAN and inter-CPAN traffic is discussed in Section V. Results of the performance analysis are presented and discussed in Section VI. Section VII concludes the paper with stating future work.

## II. CPAN and Bridge Operation

The CPAN operates under the control of a dedicated coordinator node. Time is divided into constant size superframes, the beginning and end of which are denoted with a leading and a trailing beacon frame, respectively, sent by the coordinator. Portions of the superframe subframes are devoted to beacon transmissions, data transmissions, reporting of sensing results, and sending bandwidth requests and other administrative activities.

The nodes (including the bridge) apply for bandwidth in the reservation sub-frame. Requests are sorted according to the round-robin principle, beginning with the lowest node ID that is larger than the last scheduled ID in previous superframe. The coordinator announces the pending transmissions in the leading beacon. Therefore, any given node-bridge included-from a given

CPAN must wait a random time interval with respect to the leading beacon before it can begin its transmission.

A node is allowed to request transmission only for the packets that were in its buffer at the time of the request; packets that arrive to the node during the transmission of earlier requested packets will be serviced in one of the subsequent CPAN service cycles. By the same token, the bridge also requests transmission in any given CPAN only for the inter-CPAN packets which it has received from the other CPAN. Therefore, this scheduling scheme can be modeled as a gated exhaustive policy with vacations [18].

Upon finishing the data transmission, the node has to wait for another random time period in order to synchronize with beacon. This time period, from the end of transmission to the next leading beacon (and, thus, includes the control and reservation sub-frames), is referred to as beacon synchronization.

The leading beacon also contains announcements of the sensing duty. Namely, upon successful packet transmission nodes have to 'pay' by sensing some of the channels in the working band and sending the results back to the coordinator. The duration of the sensing period is determined as the product of the sensing penalty coefficient $k_{p}$ and the number of packets transmitted since the last round of sensing duty. Performing sensing duty is a prerequisite for time slot allocation for packet transmission.

The coordinator uses sensing results to build and update the free channel table. It uses this table to select the next-hop channel - i.e., the working channel for the next superframe - among the channels least likely to be busy during that time [11]. The next-hop channel is announced in the trailing beacon, together with a number of backup channels which are used to attempt recovery in case of collision with a primary user transmission [10].

In this setup, inter-CPAN traffic is routed through the shared bridge node which collects the data from one CPAN and delivers it to the other and vice versa, as shown in the Fig. 1. To this end, the bridge hops back and forth between the CPANs without predefined rendezvous times. In each of the CPANs, the bridge must request bandwidth in order to deliver data packets and report its presence to the corresponding CPAN coordinator in order to receive data packets. However, the bridge is not present in either CPAN all the time, which means there is a risk that it might not learn about the next-hop channel and thus be unable to return to the corresponding CPAN due to dynamic channel hopping. Synchronization can be maintained if the bridge listens to every trailing beacon so as to learn about the corresponding next-hop channel. The bridge
must also listen to the leading beacon in the CPAN it is currently associated with in order to be able to send and receive data. Obviously the overhead of these actions is considerable, which is why the bridge is not required to perform sensing duty.

Transmission requests are serviced in the round-robin fashion, i.e., nodes with lower ID get access before a node with higher ID. If any of the requests can't be accommodated in the current superframe, it will be deferred to the next one. This applies to the transmissions of ordinary nodes as well as to those that involve the bridge, both as the source and as the destination. In the latter case, the coordinator can allocate bandwidth to inter-CPAN traffic (for which the bridge is the recipient) only if the bridge is present in the CPAN, which is why the bridge needs to report its presence to the coordinator. Note that the bridge, once it begins its data packet exchange with nodes in a given CPAN, will stay in that CPAN for as many superframes as necessary, not counting the visits to the other CPAN in order to listen to the trailing beacons.

## III. Modeling the MAC Algorithm

We consider a network with two CPANs, hereafter referred to as CPAN-S and CPAN-X, having $M_{S}$ and $M_{X}$ nodes, respectively, which include the coordinator node for each CPAN as well as a shared node which serves as the bridge between the CPANs. Time is measured in unit slots, while the duration of each superframe is $s_{f}$ slots. Since CPANs are formed at different times and may experience collisions with PU transmissions and subsequent recovery on different channels, their superframes are not aligned with each other. The time lag between the starting points of their respective superframes is $\alpha_{1}$ unit slots, where $\alpha_{1}$ is a random value that is uniformly distributed over one superframe. We assume that data packets have a constant size of $k_{d}$ slots with an additional slot used for acknowledgments, for a total of $k_{d}+1$ slots per packet. Let $\lambda_{S}$ and $\lambda_{X}$ denote the packet arrival rate per node for CPAN-S and CPAN-X, respectively, assuming Poisson arrivals of data packets. We also assume that $P_{i c S X}$ and $P_{i c X S}$ are the fractions of the traffic generated by each node of CPAN-S and CPAN-X, respectively for the inter-CPAN traffic (i.e., the destination is in the other CPAN). Therefore, the arrival rates for the inter-CPAN traffic will be $\lambda_{b S X}=P_{i c S X}\left(M_{S}-2\right) \lambda_{S}$ and $\lambda_{b X S}=P_{i c X S}\left(M_{X}-2\right) \lambda_{X}$, for traffic from CPAN-S to CPAN-X and vice versa, respectively.

The timing diagram in Fig. 2 depicts the general CPAN service cycle as well as the operation of an arbitrary ordinary node and the bridge node. A CPAN service cycle is defined as the time


Fig. 2. Pertaining to packet arrivals during a service cycle.
period between two successive transmissions of the same node. Due to traffic variability and sensing policy, CPAN service cycle is a random variable that can span a number of superframes. Modeling the service period. The probability generating function (PGF) for constant packet size $k_{d}$ with additional acknowledgement is $b(z)=z^{k_{d}+1}$, with the mean value of $\bar{b}=k_{d}+1$. The Laplace-Stieltjes transform (LST) of packet time, $b^{*}(s)=e^{-s\left(k_{d}+1\right)}$, can be obtained by replacing variable $z$ with $e^{-s}$. Therefore, the offered load per node is $\rho_{S}=\lambda_{S} \bar{b}$ and $\rho_{X}=\lambda_{X} \bar{b}$ for CPAN-S and CPAN-X, respectively.

The distribution of the number of packets that arrive at the ordinary node buffer between two successive transmission requests can be represented by the PGF of $\beta_{S}(z)=\sum_{k 1=1}^{\infty} \beta_{S k 1} z^{k 1}$ and $\beta_{X}(z)=\sum_{k 2=1}^{\infty} \beta_{X k 2} z^{k 2}$ for CPAN-S and CPAN-X, respectively. The $\beta_{S k 1}$ and $\beta_{X k 2}$ represent the mass probabilities that $k 1$ and $k 2$ denote the number of packets found in the buffer when an ordinary node applies for bandwidth, in the CPAN-S and CPAN-X, respectively. Therefore, mean number of packets to be transmitted in a single cycle of CPAN-S and CPAN-X are $\overline{\beta_{S}}=$ $\beta_{S}^{\prime}(1)=\sum_{k 1=1}^{\infty} k 1 \beta_{S k 1}$ and $\overline{\beta_{X}}=\beta_{X}^{\prime}(1)=\sum_{k 2=1}^{\infty} k 2 \beta_{X k 2}$, respectively.

The probability distribution of the duration of transmission (service) period for CPAN-S and CPAN-X can be represented by the PGF $S_{S}(z)=\beta_{S}(b(z))$ and $S_{X}(z)=\beta_{X}(b(z))$, respectively, as this duration depends on the number of packets that arrive during two successive transmission requests and the duration of each packet sent between the two. Therefore, the LST of the duration of ordinary node service periods are $S_{S}^{*}(s)=\beta_{S}\left(b\left(e^{-s}\right)\right)=\beta_{S}\left(b^{*}(s)\right)$ and $S_{X}^{*}(s)=$ $\beta_{X}\left(b\left(e^{-s}\right)\right)=\beta_{X}\left(b^{*}(s)\right)$; their mean values are $\overline{S_{S}}=\overline{\beta_{S} b}$ and $\overline{S_{X}}=\overline{\beta_{X} b}$, for CPAN-S and CPAN-X, respectively.

CPAN service cycle. The CPAN service cycle time is the time between two successive transmission opportunities for a given node. This time varies depending on the number of nodes and
bridge transmission time.
The PGF for the CPAN-S service cycle time can be represented by

$$
\begin{equation*}
C_{S c y c}(z)=\left(\left(1-\rho_{S}\right)+\rho_{S} S_{S}(z)\right)^{M_{S}-2} b_{e x X S}(z) \tag{1}
\end{equation*}
$$

where $b_{\text {exXS }}(z)$ is the PGF for the duration of bridge exchange in CPAN-S which depends on the number of packets the bridge has received from CPAN-X. The LST of this CPAN-S service cycle time is

$$
\begin{equation*}
C_{S c y c}^{*}(s)=\left(\left(1-\rho_{S}\right)+\rho_{S} S_{S}^{*}(s)\right)^{M_{S}-2} b_{e x X S}^{*}(s) \tag{2}
\end{equation*}
$$

with mean value of $\overline{C_{S c y c}}=-C_{S c y c}^{*^{\prime}}(0)$.
By the same token the PGF for the CPAN-X service cycle time is

$$
\begin{equation*}
C_{X c y c}(z)=\left(\left(1-\rho_{X}\right)+\rho_{X} S_{X}(z)\right)^{M_{X}-2} b_{e x S X}(z) \tag{3}
\end{equation*}
$$

where $b_{e x S X}(z)$ is the PGF for the duration of bridge exchange in CPAN-X, the LST of which is

$$
\begin{equation*}
C_{X c y c}^{*}(s)=\left(\left(1-\rho_{X}\right)+\rho_{X} S_{X}^{*}(s)\right)^{M_{X}-2} b_{e x S X}^{*}(s) \tag{4}
\end{equation*}
$$

with mean value of $\overline{C_{X c y c}}=-C_{X c y c}^{*^{\prime}}(0)$.
Bridge cycle time. Bridge cycle time is the time between two successive bridge transmissions in any given CPAN; this time depends on a number of phases as shown in the Fig. 1.

After transmitting inter-CPAN packets collected from CPAN-X in the CPAN-S, the bridge stays in the remaining superframe to receive inter-CPAN packets and to synchronize with beacons. The time from transmitting packet(s) in the current superframe to the next beacon may be considered as residual time, using the terms of renewal theory [18] where residual time is the time interval from an arbitrary moment in a renewal cycle to the beginning of the new renewal cycle, and its LST is

$$
\begin{equation*}
s_{f+}^{*}(s)=\frac{1-e^{-s s_{f}}}{s s_{f}} \tag{5}
\end{equation*}
$$

with the mean value of $\overline{s_{f+}}=-s_{f+}^{*^{\prime}}(0)$.
Then, the bridge switches over to the CPAN-X to apply for bandwidth. However, it has to wait for $\alpha_{1}$ (i.e., the time lag between superframes of CPAN-S and CPAN-X) before it can place the bandwidth request.

Due to the round robin service discipline, upon placing bandwidth request, the bridge has to wait while nodes with lower IDs are being serviced in the CPAN-X. The time from the beginning
of a new superframe to an arbitrary point in that superframe may be considered as elapsed time, using the terms of renewal theory [18] where elapsed time is the time interval from the beginning of the new renewal cycle to an arbitrary moment in that cycle, and its LST is

$$
\begin{equation*}
C_{X c y c-}^{*}(s)=\frac{1-C_{X c y c}^{*}(s)}{s \overline{C_{X c y c}}} \tag{6}
\end{equation*}
$$

with the mean value of $\overline{C_{X c y c-}}=\frac{C_{X c y c}^{(2)}}{2 \overline{C_{\text {xcyc }}}}$.
During its allocated transmission time, the bridge sends all packets it has received from CPANS. The PGF of this exchange time is $b_{e x S X}(z)$ with mean value $\overline{b_{e x S X}}$. Note that this exchange may last for several superframes.

Once the transmission is over, the bridge has to remain in CPAN-X for an additional time in order to receive packets for destinations in CPAN-S and synchronize with the beacons; the PGF of this time is $s_{f+}^{*}(s)$.

The bridge then switches back to the CPAN-S in order to deliver those packets. Due to the time lag between superframes, the bridge must wait to place the bandwidth request. The PGF of that time is $s_{f}-\alpha_{1}$ to CPAN-X.

Finally, after applying for bandwidth, the bridge waits for another round-robin waiting time in the CPAN-S; the LST of this time is

$$
\begin{equation*}
C_{S c y c-}^{*}(s)=\frac{1-C_{S c y c}^{*}(s)}{s \overline{C_{S c y c}}} \tag{7}
\end{equation*}
$$

with the mean value of $\overline{C_{S c y c-}}=\frac{C_{S c y c}^{(2)}}{2 C_{\text {Scyc }}}$.
Therefore, the total bridge cycle time for CPAN-S is

$$
\begin{equation*}
B_{S c y c}=\overline{s_{f+}}+\alpha_{1}+\overline{C_{X c y c-}}+\overline{b_{e x S X}}+\overline{s_{f+}}+\left(s_{f}-\alpha_{1}\right)+\overline{C_{S c y c-}} . \tag{8}
\end{equation*}
$$

The corresponding PGF is

$$
\begin{equation*}
B_{S c y c}(z)=z^{\left(2 \bar{s}_{f+}+s_{f}\right)} C_{X c y c-}(z) b_{e x S X}(z) C_{S c y c-}(z) \tag{9}
\end{equation*}
$$

with the LST of

$$
\begin{align*}
B_{S c y c}^{*}(s) & =B_{S c y c}\left(e^{-s}\right)  \tag{10}\\
& =e^{-s\left(2 \overline{s_{f+}}+s_{f}\right)} C_{X c y c-}^{*}(s) b_{e x S X}^{*}(s) C_{S c y c-}^{*}(s) \tag{11}
\end{align*}
$$

During this time, the number of packets collected in CPAN-X for destinations in the other CPAN can be described with the PGF of

$$
\begin{equation*}
Q b_{S}(z)=B_{S c y c}^{*}\left(\lambda_{b S X}-\lambda_{b S X} z\right) \tag{12}
\end{equation*}
$$

Since each packet needs a service time of $b(z)$, the PGF for the duration of the bridge exchange in the CPAN-X is

$$
\begin{equation*}
b_{e x S X}(z)=Q b_{S}(b(z)) \tag{13}
\end{equation*}
$$

and the LST of bridge transmission is

$$
\begin{equation*}
b_{e x S X}^{*}(s)=b_{e x S X}\left(e^{-s}\right) \tag{14}
\end{equation*}
$$

The probability distribution of the duration of bridge exchange in the CPAN-X can also be represented as a series:

$$
\begin{equation*}
b_{e x S X}(z)=\sum_{k=0}^{k_{\max }} d_{k} z^{k} \tag{15}
\end{equation*}
$$

where $d_{k}$ represents the mass probability that the bridge exchange takes $k$ slots. The mass probabilities in (15) can be obtained by expanding (13) into power series on variable $z$.

By the same token, the bridge cycle time for CPAN-X is

$$
\begin{equation*}
B_{X c y c}=\overline{s_{f+}}+\left(s_{f}-\alpha_{1}\right)+\overline{C_{S c y c-}}+\overline{b_{e x-X S}}+\overline{s_{f+}}+\alpha_{1}+\overline{C_{X S c y c-}} \tag{16}
\end{equation*}
$$

The PGF for the bridge cycle time in CPAN-X is

$$
\begin{equation*}
B_{X c y c}(z)=z^{\left(2 \bar{s}_{f+}+s_{f}\right)} C_{S c y c-}(z) b_{e x X S}(z) C_{X c y c-}(z) \tag{17}
\end{equation*}
$$

The probability distribution of the duration of bridge exchange in CPAN-S can be found in an analogous manner.

Using (14), we can define the PGF for the number of packet arrivals to an ordinary node buffer of CPAN-X during the bridge transmission as

$$
\begin{equation*}
A_{b e}(z)=b_{e x S X}^{*}\left(\lambda_{X}-\lambda_{X} z\right) . \tag{18}
\end{equation*}
$$

However, for a given CPAN, a target node may not experience bridge transmission delay in every superframe as the bridge transmission takes place once per bridge cycle period which can last several superframes. Probability that the bridge transmission takes place in the current superframe of CPAN-X is $P_{b t}=s_{f} / \overline{B_{X c y c}}$. Therefore, the final PGF for the number of packet arrivals to an ordinary node buffer of CPAN-X during the bridge transmission can be defined as

$$
\begin{equation*}
A_{b e_{p}}(z)=P_{b t} A_{b e}(z)+\left(1-P_{b t}\right) . \tag{19}
\end{equation*}
$$

Round-robin waiting time. As explained above, round-robin waiting time is the time a node has to wait while nodes with lower IDs are being serviced. Using (6), we can define the PGF of the number of packet arrivals during the round-robin waiting time as

$$
\begin{equation*}
A_{r r}(z)=C_{X c y c-}^{*}\left(\lambda_{X}-\lambda_{X} z\right) \tag{20}
\end{equation*}
$$

The number of packet arrivals during the transmission (service period) can be described by the PGF

$$
\begin{equation*}
A_{t x}(z)=S_{X}^{*}\left(\lambda_{X}-\lambda_{X} z\right) \tag{21}
\end{equation*}
$$

Delay due to beacon synchronization. The time from transmitting packet(s) in the current superframe to the next control sub-frame (at which time a node can submit the sensing report to the coordinator) may be considered as residual time of a superframe. Hence, the PGF for the number of packet arrivals during this time is

$$
\begin{equation*}
A_{s y n}(z)=s_{f+}^{*}\left(\lambda_{X}-\lambda_{X} z\right) \tag{22}
\end{equation*}
$$

Duration of sensing. The coordinator assigns sensing duty to the nodes that have transmitted their packets, and the duration of this duty, expressed in superframes, is the product of sensing penalty, $k_{p}$ and the number of packets transmitted in the service period of the corresponding CPAN ( $k_{1}$ for CPAN-S-S, $k 2$ for CPAN-X). Thus, the distribution of time spent in sensing can be represented by the PGF $V_{X}(z)=\sum_{k 2=1}^{\infty} \beta_{X k 2} z^{k_{p} s_{f} k 2}=\beta_{X}\left(z^{k_{p} s_{f}}\right)$ and its mean value is $\overline{V_{X}}=k_{p} s_{f} \overline{\beta_{X}}$. The corresponding LST of the single sensing period is $V_{X}^{*}(s)=\sum_{k 2=1}^{\infty} \beta_{X k 2} e^{-k_{p} s_{f} k 2}$. The number of packets that arrive during the sensing period can be obtained by replacing $s$ with $\lambda_{X}-\lambda_{X} z$ in the last equation,

$$
\begin{equation*}
A_{X v c}(z)=V_{X}^{*}\left(\lambda_{X}-\lambda_{X} z\right) \tag{23}
\end{equation*}
$$

Impact of packet reception during sensing. As explained above, reception preempts sensing in the sense that a node has to takes a break from ongoing sensing duty in order to receive packets. Packet reception may take place in one or more superframes, but the node still has to finish its sensing duty before placing a new transmission request. As the result, the sensing period will be effectively expanded due to reception. To model this effect, we need to find the probability of packet reception during the sensing period.

Each of the nodes in the CPAN-X generates intra-CPAN (local) traffic at a rate of $\frac{\lambda_{X}\left(1-P_{i c}\right)}{M_{X}-2}$, assuming uniform distribution of destination nodes. Probability of having no packets for a given
target node during a sensing period is $P_{n r s}=e^{-\frac{\lambda_{X}\left(1-P_{i c}\right)}{M_{X}-2}} \overline{V_{X}}$, and the PGF for extended sensing period due to reception is $V_{X e s}(z)=P_{n r s}+\left(1-P_{n r s}\right) z^{s_{f}}$.

However, a node in the CPAN-X receives intra-CPAN traffic at a rate of $\frac{\lambda_{X}}{M_{X}-2}$, and interCPAN packets at a rate of $\frac{\lambda_{b S X}}{M_{X}-2}$. Probability of having no packets during the sensing period is $P_{n r d}=e^{-\frac{\lambda_{b S X}}{M_{X}-2} \overline{B_{X c y c}}} e^{-\frac{\lambda_{X}}{M_{X}-2} \overline{V_{X}}}$. Therefore, the PGF for extended sensing period in the CPAN-X due to reception is $V_{X e s}(z)=P_{n r d}+\left(1-P_{n r d}\right) z^{s_{f}}$. The number of packets that arrive during this extended sensing period can be described by the PGF of $A_{X v e s}(z)=V_{X e s}\left(\lambda_{X}-\lambda_{X} z\right)$.

Time between successive transmission requests. The PGF for the number of packets arrivals to a node in the CPAN-X during the interval between two successive bandwidth requests is

$$
\begin{equation*}
Q_{X}(z)=A_{b e_{p}}(z) A_{r r}(z) A_{t x}(z) A_{s y n}(z) A_{X v e s}(z)=\sum_{k 2=0}^{\infty} q_{X k 2} z^{k 2} \tag{24}
\end{equation*}
$$

If the node finishes sensing and has no packets in its buffer, it will continue with sensing duty, which occurs with the probability of $q_{X 0}=Q_{X}(0)$. The distribution of the number of packets that arrive at the ordinary node buffer between two successive transmission requests can be described by the PGF of

$$
\begin{equation*}
\beta_{X}(z)=\frac{Q_{X}(z)-q_{X 0}}{1-q_{X 0}} \tag{25}
\end{equation*}
$$

and its mean value is $\overline{\beta_{X}}=\bar{A}_{b e_{p}}+\bar{A}_{r r}+\bar{A}_{t x}+\bar{A}_{s y n}+\bar{A}_{X v e s}$. Probability distribution of the total sensing period is

$$
\begin{equation*}
V_{X t o t}(z)=\frac{V_{X e s}\left(1-q_{X 0}\right)}{1-V_{X e s} q_{X 0}} \tag{26}
\end{equation*}
$$

Equations from (1) to (26) and the corresponding equations from CPAN-S which can be derived in an analogous manner can be solved together as a system with unknowns $\beta_{S k 1}$ and $\beta_{X k 2}$ for $k 1=k 2=1 \ldots n_{c}$, if we limit the number of terms in each PGF or LST to $n_{c}$.

## IV. Probability of collision with primary source

An ongoing node's transmission in cognitive network may collide with primary source. There are two cases when node's transmission is destroyed by the activity of primary source:
(a) The coordinator can select a wrong channel (primary source is already active on that channel) if the channel table has inaccurate status about that channel.
(b) The primary source becomes active on the channel which is currently being used by the node for transmission.

For the first case, the coordinator creates and updates the channel status table by using the channel sensing results. The nodes are assigned to sense the channels periodically and report the channel status to the coordinator. If the duration between two consecutive sensing events on a single channel increases, the probability that the channel is incorrectly observed increases. This is due to the fact that the primary source may have higher chance to change its state from idle to active or vice versa during these longer sensing events. Thus the probability of inaccurate channel status in the channel table will increase while smaller number of nodes are performing sensing compared to the number of channels. On the other hand the second case may happen at any time since the primary source may change its state from idle to active during ongoing superframe.

Therefore, total collision probability for bridge node as well as other ordinary nodes can be calculated by adding the probability of inaccurate channel status in the channel table and the probability of collision during transmission.

In order to model the collision probability with primary source we consider that:
(i) Each primary source is intermittently active on its own channel. The probability density function (pdf) of ON and OFF periods are $g_{o n}(x)$ and $g_{o f f}(x)$ and mean values are $\bar{G}_{o n}$ and $\bar{G}_{\text {off }}$ respectively. Therefore the mean value of total cycle time of primary source activity is $\bar{G}_{c y l}=\bar{G}_{o n}+\bar{G}_{o f f}$ and the activity probability of primary source is $p_{o n}=\frac{\bar{G}_{o n}}{\bar{G}_{c y c}}$.
(ii) The CPAN superframe starts from a random point in the idle (OFF) channel period and continues until it finishes or the channel becomes active ( $\mathrm{ON} \mathrm{)} \mathrm{due} \mathrm{the} \mathrm{presence} \mathrm{of} \mathrm{primary}$ source. In the later case the superframe collides with primary source. However, we need to find the residual channel idle time in order to find successful or unsuccessful transmission. The probability density function (pdf) of residual channel idle time is proportional to the probability that channel idle time is larger than some value $y$. This pdf can be represented as follows: $f(y)=\frac{\int_{z=y}^{\infty} g_{o f f}(z) d z}{\overline{G_{o f f}}}$. Therefore the probability distribution function (PDF) of residual channel idle time is $F(x)=\int_{0}^{x} d(y) d y$.

Let us now find the collision probability for CPAN-X's nodes. Once we find it for CPAN-X's nodes, the collision probability for CPAN-S's nodes can be found in an analogous manner.

## A. Probability that coordinator has inaccurate channel information

The probability that an ordinary node from CPAN-X is performing channel sensing is

$$
\begin{equation*}
P_{s}=\frac{\overline{V_{X t o t}}}{\overline{C_{X c y c-}}+\overline{S_{X}}+\frac{s_{f}}{\overline{B_{X c y c}}}\left(\overline{b_{e x S X}}\right)+\overline{s_{f+}}+\overline{V_{X t o t}}} \tag{27}
\end{equation*}
$$

Therefore, the probability distribution of the number of nodes in CPAN-X concurrently performing channel sensing has the PGF

$$
\begin{equation*}
\Theta(z)=\sum_{i=0}^{M_{X}-1}\binom{M_{X}-1}{i} P_{s}^{i}\left(1-P_{s}\right)^{M_{X}-1-i} z^{i}=\sum_{n=0}^{M_{X}-1} \theta_{n} z^{n} \tag{28}
\end{equation*}
$$

where $\theta_{n}$ represents the mass probability that $n$ nodes are performing channel sensing simultaneously. Note that the coordinator performs the sensing job itself when no node is available for doing the sensing.

In order to find the probability distribution of two consecutive sensing events we also assume a node jumps over channels in every $d$ unit slots. While node performing sensing it jumps randomly over all channels except the channels which are currently occupied by both CPANs, i.e., $N-2$ channels. Therefore, the probability that a node is coming to a particular channel for sensing is $P_{j}=\frac{1}{N-2}$. By using Eq. 28 and the above mentioned assumption we can find the PGF for the time period between two consecutive sensing events on a particular channel:

$$
\begin{align*}
\Omega(z) & =\theta_{0} \sum_{k=1}^{\infty} P_{j}\left(1-P_{j}\right)^{k-1} z^{k d} \\
& +\sum_{l=1}^{\min \left(M_{X}-1, N-3\right)} \theta_{l} \sum_{k=1}^{\infty} l P_{j}\left(1-l P_{j}\right)^{k-1} z^{k d} \\
& +\sum_{\min \left(M_{X}-1, N-3\right)+1}^{M_{X}-1} z^{d} \tag{29}
\end{align*}
$$

By applying renewal theory [18] and the steps followed in [8] we can calculate the probability $P_{i}$ of having inaccurate channel status and its duration in the channel table.

## B. Probability of collision during transmission

Since the primary source becomes active in any moment, we need to find the probability that channel will become active during ongoing node's transmission. The CPAN's superframe starts at random point in the idle channel period. Thus, the collision takes place when the residual
channel idle time is less than the superframe duration. Therefore, the collision probability of bridge transmission and ordinary node transmission in CPAN-X can be calculated as follows:

$$
\begin{gather*}
P_{c}^{(b r)}=\int_{x=0}^{\infty}\left(D\left(x+\overline{C_{X c y c-}}+\overline{b_{e x S X}}\right)-D(x)\right) d(x) d x  \tag{30}\\
P_{c}^{(o)}=\int_{x=0}^{\infty}\left(D\left(x+\overline{C_{X c y c-}}+\overline{S_{X}}\right)-D(x)\right) d(x) d x \tag{31}
\end{gather*}
$$

However, nodes miss the next channel announcement (by missing the trailing beacon) when transmission is destroyed by the activity of primary source. In this case recovery procedure as described earlier will take place on the channels which are announced as backup channels in the previous trailing beacon.

## V. Packet access delay

Intra-CPAN packet delay. Let us assume that an ordinary node in the CPAN-X already has $L^{*}$ intra-CPAN packets at the moment it applies for bandwidth. Let us also assume that $A_{i}$ packets arrive to the node while it is transmitting the $i^{\text {th }}$ packet. Thus after transmitting the $n^{\text {th }}$ packet in the transmission (service) period, the buffer has $L_{n}=L^{*}+A_{r r}+A_{1}+A_{2}+\ldots+A_{n}-n$ packets, and the PGF of the number of packets left after $n^{\text {th }}$ departing packet can be obtained as

$$
\begin{equation*}
L_{n}(z)=\frac{A_{r r}(z) A(z)^{n} \sum_{k=n}^{\infty} \beta_{X k} z^{k}}{z^{n} \sum_{k=n}^{\infty} \beta_{X k}} \tag{32}
\end{equation*}
$$

The PGF of the number of packets left in the buffer after any departing packet can be deduced from the last equation and the single packet serving time $b^{*}(\lambda-\lambda z)$ as

$$
\begin{align*}
L(z) & =\sum_{n=1}^{\infty} \frac{\sum_{k=n}^{\infty} \beta_{X k}}{\overline{\beta_{X}}} L_{n}(z) \\
& =A_{r r}(z) \frac{\left(\beta_{X}\left[b^{*}\left(\lambda_{X}-\lambda_{X} z\right)\right]-\beta_{X}(z)\right) b^{*}\left(\lambda_{X}-\lambda_{X} z\right)}{\overline{\beta_{X}}\left[b^{*}\left(\lambda_{X}-\lambda_{X} z\right)-z\right]} \tag{33}
\end{align*}
$$

Packets are serviced in FIFO order and the number of packets left after a departing packet is equal to the number of packets that arrived during the departing packet was in the system. Probability distribution of the packet waiting time can be obtained from

$$
\begin{equation*}
L(z)=T_{a}^{*}\left(\lambda_{X}-\lambda_{X} z\right)=W^{*}\left(\lambda_{X}-\lambda_{X} z\right) b^{*}\left(\lambda_{X}-\lambda_{X} z\right) \tag{34}
\end{equation*}
$$

and the corresponding LST of intra-CPAN packet waiting time is

$$
\begin{equation*}
W^{*}(s)=A_{r r}\left(1-\frac{s}{\lambda_{X}}\right) \frac{\lambda_{X}\left(\beta_{X}\left[b^{*}(s)\right]-\beta_{X}\left(1-\frac{s}{\lambda_{X}}\right)\right)}{\overline{\beta_{X}}\left[\lambda_{X} b^{*}(s)-\lambda_{X}+s\right]} \tag{35}
\end{equation*}
$$

with the mean value of $\bar{W}=\left.\frac{d W^{*}}{d s}\right|_{s=0}=\frac{\left(1+\rho_{X}\right) \beta_{X}^{(2)}(1)}{2 \lambda_{X} \overline{\beta_{X}}}$.
Inter-CPAN packet delay. We note that an inter-CPAN packet undergoes no less than five different phases from its arrival to the CPAN-S to the reception by the CPAN-X node and vice versa, as shown in Fig. 1.

First, the packet waits to be transmitted to the bridge. This time, commonly referred to as access delay, $W_{a}$, is equal to the residual time of bridge cycle period, $W_{a}^{*}(s)=\frac{1-B_{S c y c}^{*}(s)}{s \bar{B}_{\text {Scyc }}}$, with mean value of $\bar{W}_{a}=-W_{a}^{*^{\prime}}(0)$.

Second, the packet waits until bridge finishes receiving all of the inter-CPAN packets from the CPAN-S and synchronizes with the beacon, which lasts for the remaining superframe duration, $W_{s y n}^{*}(s)=\frac{1-e^{-s s_{f}}}{s s_{f}}$, with mean value of $\bar{W}_{s y n}=-W_{s y n}^{*^{\prime}}(0)$.

Third, the bridge switches to CPAN-X to apply for bandwidth. Due to time lag $\alpha_{1}$ between CPAN-S and CPAN-X, the bridge waits for $W_{a b}=\alpha_{1}$ before applying for bandwidth in the CPAN-X.

Fourth, bridge waits for its turn to transmit in CPAN-X for round-robin waiting time. The delay observed by an inter-CPAN packet may thus be considered to be the elapsed time of the duration of CPAN-X cycle time, $W_{r r}^{*}(s)=\frac{1-C_{X c y c}^{*}(s)}{s \bar{C}_{X c y c}}$, with mean value of $\bar{W}_{r r}=-W_{r r}^{*^{\prime}}(0)$.

Finally, the bridge stays in CPAN-X to deliver its packets to target node. Since packets are randomly positioned within the bridge queue, the delay experienced by a bridge packet in the CPAN-X is the elapsed time of the duration of bridge transmission: $W_{b t}^{*}(s)=\frac{1-b_{e x S X}^{*}(s)}{s \bar{b}_{e x S X}}$, the mean value of which is $\bar{W}_{b t}=-W_{b t}^{*^{\prime}}(0)$.

Therefore, mean end-to-end delay for an inter-CPAN packet from CPAN-S to CPAN-X can be obtained as $\bar{W}_{a}+\bar{W}_{s y n}+W_{a b}+\bar{W}_{r r}+\bar{W}_{b t}$.

The mean end-to-end delay for an inter-CPAN packet from CPAN-X to CPAN-S can be found in an analogous manner, except that the bridge waits $W_{a b}=s_{f}-\alpha_{1}$ before applying for bandwidth in CPAN-S.

## VI. Performance Analysis

To evaluate the performance of the proposed scheme, we have solved the system of equations presented above using Maple 16 from Maplesoft, Inc. [7]. The number of nodes for CPAN-S is $M_{S}=14$ and CPAN-X is $M_{X}=10$; these number include the coordinator in each CPAN and a shared bridge node. The number of nodes of each CPAN impacts the flow of inter-CPAN traffic towards the other CPAN; the higher the number of nodes, the higher the number
of inter-CPAN packets is generated in that CPAN. We have assumed that both CPANs use $N=19$ channels, each with an independent primary source. However, the number of channels for each CPAN gets for its operation depends on that CPAN's geographical position in the standard operating frequency range $(54-862 \mathrm{MHz})$ and the bandwidth $(5-8 \mathrm{MHz})$ of each channel [16]. By considering a capacity of about 19.8 Mbps per 6 MHz channel, we have assumed each packet size is 1 Kbyte . The packet needs $k_{d}=10$ time units to be transmitted with an additional acknowledgement duration to one time unit when the time unit is set to a single sensing slot, $d=1$. Mean cycle time of primary source has been set to $G_{c y l}=3000$ time units where ON, $G_{o n}$ and OFF, $G_{o f f}$ periods are exponentially distributed with mean values of 900 and 2100 time units respectively. The size of the superframe $s_{f}$ is set to 130 time units, 30 of which are allocated for the control and reservation purposes. In order to keep a tolerable collisions with primary sources, the transmission duration of each superframe is set around $\mathbf{1 5 \%}$ of the transmission duration of each primary source. Packet arrival rate varies from $\lambda_{S}=0.001$ to $\lambda_{S}=0.006$ in CPAN-S and $\lambda_{X}=0.001$ to $\lambda_{X}=0.006$ in CPAN-X. Sensing penalty was set to $k_{p}=0.6$ while the time lag $\alpha_{1}$ was initially set to $0.5 s_{f}=65$ time units.

## A. CPANs performance while exchanging inter-CPAN traffic

In our first experiment we have examined the behaviour of each CPAN while bridge exchanging packets. This experiment has been conducted by varying packet arrival rate to both CPANs while the fractions of inter-CPAN traffic were set to $P_{i c S X}=0.2$ and $P_{i c X S}=0.2$. CPANs.

Fig. 3 shows the performance of ordinary node operations: mean number of packet arrivals to an ordinary node between successive transmissions by that same node and mean service cycle time - i..e, the time needed to service all nodes in a CPAN.

As expected, the number of packet arrivals increases with the increase in the traffic intensity of the local CPAN. However, it also increases with the increase in traffic intensity of the other CPAN, which is due to the fact that the duration of bridge exchange of a given CPAN depends on the traffic intensity in the other CPAN. As the result, bridge exchanges last longer and more packets arrive to the node during that time. As can be seen, mean number of packet arrivals begins at approximately equal values in both CPANs, but the maximum value is higher in CPANX, Fig. 3(c), than in CPAN-S, Fig. 3(a). This is caused by the higher number of inter-CPAN packets sent from CPAN-S which has more nodes; this translates into longer bridge exchanges in CPAN-X and, consequently, longer intervals between successive packet transmissions.
 CPAN-X.

Fig. 3. Mean number of packet arrivals to an ordinary node between two successive transmissions (top row) and mean service cycle time (bottom row).

Regarding the service cycle, we observe that higher traffic intensity translates into higher number of packets and, by extension, into longer service cycle period. Higher traffic intensity also means that the volume of inter-CPAN traffic will be higher and bridge transmissions will last longer, which should also contribute to the extension of the service cycle. Due to the interdependency of the CPANs introduced by the bridge, mean service cycle time also increases for higher traffic intensity in the other CPAN.

Fig. 4 shows the parameters of bridge operations: mean duration of a bridge exchange and mean bridge cycle period. Mean duration of bridge cycle period depends on the duration of the bridge exchange in both CPANs as well as on the round-robin waiting time. As can be


Fig. 4. Mean duration of bridge exchange and mean bridge cycle period.
seen, mean bridge cycle period increases at higher traffic intensity, with longer values observed in CPAN-S which has more nodes and, consequently, generates more inter-CPAN traffic. This leads to an increase of the bridge cycle in CPAN-X and extends the waiting time for traffic in that CPAN, both local and non-local, which in turn contributes to the increase of bridge cycle in CPAN-S.

Regarding bridge exchange, the higher number of nodes in CPAN-S means that a higher number of inter-CPAN packets are generated there. To deliver those packets to their destinations in CPAN-X, the bridge exchange must last longer in CPAN-X, as can be seen by comparing the diagram in Fig. 4(c) with the one in Fig. 4(a).

Fig. 5 shows the mean packet waiting times or delays, first for intra-CPAN (local) traffic in each CPAN, then for inter-CPAN (non-local) traffic. Of course, the intra-CPAN packet waiting

(a) Mean intra-CPAN traffic delay in CPAN-S.

(b) Mean intra-CPAN traffic delay in CPAN-X


Fig. 5. Mean end-to-end delay.
time increases with traffic intensity in that CPAN. However, the intra-CPAN packet waiting time strongly depends on both the round-robin waiting time and the duration of bridge exchange in the current CPAN. The round-robin waiting time is affected by the CPAN traffic intensity, but also by the duration of bridge exchange which, in turn, depends on the traffic intensity in the other CPAN. Thus the intra-CPAN packet waiting time for a given CPAN increases for higher traffic intensity in either CPAN. Longer waiting times observed in CPAN-X, Fig. 5(b), compared to those in CPAN-S, the Fig. 5(a), are due to the longer duration of bridge exchanges in the former.

Figs. 5(c) and 5(d) show the delay for inter-CPAN packet in the direction from CPAN-S to

CPAN-X, and from CPAN-X to CPAN-S, respectively. This delay is mainly dependent on the bridge cycle period which, in turn, depends on the number of nodes in both CPANs and the packet arrival rate. This causes the bridge to carry more packets which affects bridge delivery (in the target CPAN) more than bridge collection (in the source CPAN). As the result, inter-CPAN delays turn out to be longer for traffic from CPAN-S to CPAN-X, Fig. 5(c), than for traffic going in the opposite direction, Fig. 5(d).

Our final scenario of this part of experiment concerns the mean end-to-end delay for interCPAN packet as the function of the time lag between CPAN superframes when packet arrival rates in both CPANs are set to $\lambda_{S}=\lambda_{X}=0.003$. In this scenario, the delay is affected by the time lag only, in the phase where the bridge is waiting to apply for bandwidth in the destination CPAN, while all other components of the end-to-end delay remain unchanged. Therefore, end-toend delay for inter-CPAN packets increases almost linearly with the increase of time lag, $\alpha_{1}$ in case of the traffic from CPAN-S to CPAN-X, or $s_{f}-\alpha_{1}$, in the case of the traffic from CPAN-X to CPAN-S. This dependency is easily observed in Fig. 5(e). The crossover point where the two delays have equal values is not in the middle of the range for $\alpha_{1}$ on account of the difference in the number of nodes (and the resulting traffic) in the two CPANs.

## B. The impact of other CPAN's traffic and fraction of inter-CPAN packet

In this experiment we have examined the behaviour of any given CPAN by varying the packet arrival rate of the other CPAN and the fraction of inter-CPAN traffic from the other CPAN to the observed CPAN. However, in order to analyse any CPAN performance based on the other CPAN's traffic load only we keep the local packet arrival rate and the fraction of inter-CPAN traffic fixed to 0.003 and 0.2 ,respectively.

Fig. 6 shows the mean number of packets arrival to the ordinary node buffer during two successive transmissions in any given CPAN. Though local traffic is kept constant, the average packet arrival increases slightly with the higher intensity of the other CPAN traffic and the higher fraction of inter-CPAN traffic towards a given CPAN. This is due to the fact that the duration of bridge exchange of any given CPAN depends on the other CPAN's traffic intensity and the fraction of inter-CPAN traffic. Therefore, more packets arrives to the ordinary node buffer during the extended duration of bridge exchange.

Fig. 7 shows the mean service cycle time for any given CPAN nodes. In order to serve higher number of packets due to higher traffic arrival to the ordinary nodes buffer, as can be seen in

(a) mean number of packets arrival in (b) mean number of packets arrival in CPAN-S while $\lambda_{S}=0.003$ and $P_{i c S X}=$ CPAN-X while $\lambda_{X}=0.003$ and $P_{i c X S}=$ 0.02 0.02

Fig. 6. mean number of packets that arrive to ordinary node buffer during two successive transmissions in CPAN-S and CPAN-X


> (a) mean cycle time in CPAN-S while $\lambda_{S}=($ b) mean cycle time in CPAN-X while 0.003 and $P_{i c S X}=0.02 \quad \lambda_{X}=0.003$ and $P_{i c X S}=0.02$

Fig. 7. mean service cycle time (to serve all nodes requests) in CPAN-S and CPAN-X

Fig. 6, any given CPAN node experiences longer cycle period. The mean service cycle also, more importantly while keeping the local traffic fixed, increases due to the extended bridge transmission. However, the bridge transmission depends on the number of inter-CPAN packets that the bridge collected from other CPAN. Thus, mean service cycle time also strongly depends on the traffic intensity of other CPAN as well as the fraction of inter-CPAN packets.

Fig. 7(b) shows the mean cycle time is higher compared to the Fig. 7(a). This is because the


Fig. 8. mean bridge cycle period in CPAN-S and CPAN-X

CPAN-X gets higher number of inter-CPAN packets from CPAN-S due to CPAN-S has higher number of nodes. In order to transmit this higher number of inter-CPAN packets the duration of bridge exchange in CPAN-X gets longer; thus the mean cycle time gets increasing.

Fig. 8 shows the mean bridge cycle periods. The bridge cycle period of any given CPAN changes with the duration of bridge exchange in other CPAN and the round-robin waiting time for both CPANs. As the local traffic is kept constant, the bridge cycle period only depends on the round-robin waiting time in the other CPAN. Therefore, bridge cycle period increases for higher traffic intensity of other CPAN and higher fraction of inter-CPAN traffic.

Fig. 9 shows the mean durations of bridge exchange. As expected, due to higher number of nodes, the CPAN-S generates higher number of inter-CPAN packets with the higher traffic intensity and higher fraction of inter-CPAN traffic. In order to transmit those packets the bridge takes longer exchange duration in CPAN-X, as shown in Fig. 9(b). However, as CPAN-X is smaller in size thus produces less number of inter-CPAN packets compared to CPAN-S, the bridge exchange duration in CPAN-S is shorter compared to CPAN-X, as shown in Fig. 9(a).

Fig. 10 shows the mean end-to-end delay for inter-CPAN packet. Fig. 10(a) shows the delay for inter-CPAN packet from CPAN-S to CPAN-X and Fig. 10(b) show this delay for inter-CPAN packet from CPAN-X to CPAN-S. As the time lag is constant here, this delay depends on bridge cycle period. Moreover, the end-to-end delay increases while the randomly positioned interCPAN packet waits longer in the bridge queue to deliver in the other CPAN. This is happened

(a) mean bridge transmission in CPAN-S
(b) mean bridge transmission in CPAN-X while $\lambda_{S}=0.003$ and $P_{i c S X}=0.02$ while $\lambda_{X}=0.003$ and $P_{i c X S}=0.02$

Fig. 9. mean duration of bridge exchange in CPAN-S and CPAN-X

(a) mean end-to-end delay for inter-CPAN (b) mean end-to-end delay for inter-CPAN packet from CPAN-S to CPAN-X while packet from CPAN-X to CPAN-S while $\lambda_{S}=0.003$ and $P_{i c S X}=0.02 \quad \lambda_{X}=0.003$ and $P_{i c X S}=0.02$

Fig. 10. mean ene-to-end delay for inter-CPAN traffic
when the bridge carries higher number of packets due to other CPAN higher traffic intensity and higher fraction of inter-CPAN traffic.

## C. Impact of collisions with primary source

In this experiment we have investigated how the CPAN performance is affected by the activity of primary sources. The transmission of bridge and/or ordinary nodes may be destroyed due to having inaccurate channel state stored in the channel table or primary source becomes active on

(a) average number of nodes performing (b) average number of slots between (c) probability of inaccurate channel stasensing in CPAN-S sensing events on a channel in CPAN-S tus in the channel table of CPAN-S

(d) average number of nodes performing sensing in CPAN-X
(e) average number of slots between (f) probability of inaccurate channel stasensing events on a channel in CPAN-X tus in the channel table of CPAN-X

Fig. 11. Average number of sensing nodes and number of slots between sensing events, and probability of inaccurate channel status in the channel table.
the channel where the secondary transmission is going on. The first case depends on how often a channel is sensed and the second case depends on the duration of transmission.

Fig. 11 shows the average number of nodes that perform sensing, average time between two successive sensing events and the probability of inaccurate channel status in the channel table.

Fig. 11(a) and 11(d) show that majority of the nodes engaged in sensing when the corresponding CPAN's traffic intensity is low. This is due to the nodes have no packets to transmit after getting back from regular sensing thus they go for further sensing. As the local traffic increases the number of sensing nodes decreases. This is caused by the nodes are busy with transmission and have less time to do extended sensing. On the other hand the number of sensing nodes goes down slightly when non-local traffic increases. This is due to the longer bridge transmission;
the ordinary nodes get more packets to transmit during this longer bridge transmission which causes them busy with transmission and have less time for extended sensing. As expected, the higher number of nodes are sensing simultaneously in CPAN-S, Fig. 11(a) compared to CPAN-X, Fig.11(d). This is due to the larger size of CPAN-S.

Fig. 11(b) and 11(e) show that the average time between two consecutive sensing events on a channel is inversely proportional to the number of nodes perform sensing. This is because the time between two sensing events on a single channel gets longer when inadequate number of nodes compared to the number of channels are available to perform sensing.

The probabilities of inaccurate channel status in channel table in Fig. 11(c) and 11(f) are directly proportional to the time between two consecutive sensing events on a single channel in Fig. 11(b) and 11(e) respectively. Since the more the time between two consecutive sensing events grow on a single channel the more chances of the coordinator having wrong status about that channel.

Fig. 12 shows the collision probability with primary source for bridge node and ordinary node. The probabilities of bridge collisions in Fig. 12(a) and 12(c) are higher than the probabilities of node collisions in Fig. 12(b) and 12(d) respectively since bridge transmissions are longer compared to node transmission. However, the node transmission in CPAN-X suffers from more collision, Fig. 12(d), compared to the node collisions in CPAN-S, Fig. 12(b). This is caused by the CPAN-X having less number of nodes compared to CPAN-S; this creates higher probability that CPAN-X has inaccurate channel status in the channel table. Moreover, the bridge transmission in CPAN-X suffers from more collision, Fig. 12(c), compared to the bridge collisions in CPAN-S, Fig. 12(a). This is due to the bridge takes longer transmission time in the CPAN-X since it brings higher number of packets from CPAN-S.

## VII. Conclusion and Future work

We have discussed a simple yet effective scheme for bridging between two cognitive personal area networks (CPANs). The main feature of the proposed bridging scheme is that the bridge is free to remain in a given CPAN as long as it takes to deliver all the data originating from the other CPAN and to collect all the data to be delivered there, without a predefined schedule which means that CPANs need not be synchronized with each other. Through probabilistic analysis and renewal theory, we have obtained complete probability distribution for service cycle time, bridge cycle time, and end-to-end packet delays for both intra- and inter-CPAN traffic. Experiments have

(b) probability of ordinary node collision in
(a) probability of bridge collision in CPANS CPAN-S


(c) probability of bridge collision in CPAN-
(d) probability of ordinary node collision in X

CPAN-X

Fig. 12. Collision probability with primary source
confirmed the validity of the scheme, but they have also shown that the CPANs are not fully decoupled, as the performance of one of the CPANs depends on local as well as non-local traffic intensity. Moreover, in order to evaluate the reliability of node or bridge transmission in each CPAN we have estimated the probability of collision with primary source. Experiments have also shown that bridge transmissions have higher colliding probabilities compared to ordinary node transmissions since bridge transmissions are longer compared to node transmissions. However, the collision probabilities for node as well as bridge transmission are very marginal until the traffic intensity reaches to extreme high.

Our future work will focus on extending the scheme to networks consisting of three or more CPANs, where a number of bridges can coexist and carry the inter-CPAN traffic by visiting

CPANs in sequence. The application of the proposed scheme to traditional as well as delaytolerant networks will also be studied.

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